76[K, X].—RICHARD BELLMAN (Editor), *Mathematical Optimization Techniques*, University of California Press, Berkeley and Los Angeles, California, 1963, xii + 346 p., 23 cm. Price $8.50.

The papers appearing in this volume were presented at the Symposium on Mathematical Optimization Techniques held in Santa Monica, California, October 18–20, 1960. The first four chapters by Miele, Dergabedian and Ten Dyke, Breakwell, and Dreyfus concern themselves with optimizing performance of aircraft, rockets, and problems of guidance. In Chapter 5 Parzen describes a new approach to the synthesis of optimal smoothing and prediction systems. Adaptive matched filters are discussed by Kailath in Chapter 6 and statistical communication theory by Middleton in Chapter 7. Hall in Chapter 8 presents a theory of minimum-bias estimators analogous to minimum-risk theory and points to the need for some compromise in design between bias and risk. Optimal replacement rules based on periodic inspection are considered by Derman in Chapter 9. The simplex method of linear programming and its relation to theory is examined by Tucker in Chapter 10. Wolfe, in Chapter 11, surveys computational procedures of nonlinear programming. Kruskal presents in Chapter 12 a theorem on the number of simplices in a complex. Chapter 13 consists of a discussion by Prager of optimal structural design based on plastic analysis. Recent developments in the theory of experimental design are outlined by Elfving in Chapter 14. Generalities on automation and control in the Soviet Union are given by the reviewer in Chapter 15. In Chapter 16 Kalman discusses the problem of optimal control from the Hamiltonian point of view. In the last chapter Bellman formulates the making of mathematical models as an adaptive control process.

This impressive volume is an excellent source of information on and a guide to recent developments in optimization. It covers with authority a wide range of topics.

JOSEPH P. LASALLE

RIAS
7212 Bellona Avenue
Baltimore, Maryland

77[K, Z].—ROBERT D. PHILLIPS, *Tables Useful in Statistics and Information Theory*, Federal Systems Division, International Business Machines Corporation, Rockville, Maryland, 5 November 1963, iii + 174 p. (spiral bound).

The four tables comprising the body of this report are intended for use in statistics and information theory.

Table I gives to 5D the channel capacity in bits and the associated maximizing input probabilities for a general binary channel. If the binary channel is characterized by the noise matrix (conditional probabilities)

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

then the capacity C and input probability p_1^* are tabulated for $p_{11} = 0.00000$, 0.00001, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.50, and $p_{21} = 0.00001$, 0.00005, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.99, 0.995, 0.999, 0.9995, 0.9999, 0.99995, provided $p_{11} \neq p_{21}$. This table represents an extension of one by Sakaguchi [1].

Table II consists of 5D values of $-\ln p$, $-p \ln p$, $-p \ln p - (1-p)\ln(1-p)$, $-(1-p)\ln(1-p)$, and $-\ln(1-p)$ for $p = 0.001(0.001)0.500$. This represents an extension of certain portions of a table by Bartlett [2].

Table III gives 5D values of

$$p_1 \ln (p_1/p_2),\ (1 - p_1) \ln [(1 - p_1)/(1 - p_2)],$$
$$p_1 \ln (p_1/p_2) + (1 - p_1) \ln [(1 - p_1)/(1 - p_2)],$$
$$p_1 \ln [p_1/(1 - p_2)],\ (1 - p_1) \ln [(1 - p_1)/p_2],$$

and

$$p_1 \ln [p_1(1 - p_2)] + (1 - p_1) \ln [(1 - p_1)/p_2]$$

for p_1, p_2 = 0.00001, 0.00005, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.50. This represents an extension of a table by the reviewer [3].

Table IV gives 5D values of

$$(p_1 - p_2) \ln (p_1/p_2),$$
$$(p_2 - p_1) \ln [(1 - p_1)/(1 - p_2)],$$
$$(p_1 - p_2) \ln (p_1/p_2) + (p_2 - p_1) \ln [(1 - p_1)/(1 - p_2)],$$
$$[p_1 - (1 - p_2)] \ln [p_1/(1 - p_2)],\ [(1 - p_1) - p_2] \ln [(1 - p_1)/p_2],$$

and

$$[p_1 - (1 - p_2) \ln [p_1/(1 - p_2)] + [(1 - p_1) - p_2] \ln [(1 - p_1)/p_2]$$

for the same values of p_1 and p_2 as in Table III.

All these tables were computed on an IBM 709 system, using floating-point double-precision arithmetic. According to the author, a comparison with the corresponding logarithms published by the National Bureau of Standards [4] revealed that the error in the logarithm routine used in the construction of the present tables did not exceed $3 \cdot 10^{-8}$ for arguments in the range (0.02, 0.99999) and was at most $9 \cdot 10^{-8}$ for arguments in the range (0.00001, 0.02). This instills considerable confidence in the reliability of these 5D tables.

Examples illustrating the use of the tables appear in the accompanying explanatory text, to which is appended a bibliography consisting of eight titles.

S. Kullback

Statistics Department
The George Washington University
Washington, D. C.

1. Minoru Sakaguchi, "Table for the capacity of binary communication channels," *J. Operations Res. Soc. Japan*, v. 4, 1962, p. 55–66.
2. M. S. Bartlett, "The statistical significance of odd bits of information," *Biometrika*, v. 39, 1952, p. 228–237.
3. S. Kullback, *Information Theory and Statistics*, John Wiley and Sons, Inc., New York, 1959.
4. National Bureau of Standards, *Table of Natural Logarithms for Arguments between Zero and Five to Sixteen Decimal Places*, Applied Mathematics Series 31, U. S. Government Printing Office, Washington 25, D. C.